$$P_{\max} = \frac{EV_{\infty}}{X_d^{'} + X_T + X_L}, \; at \;\; \delta = 90^{\circ} \tag{2.2}$$

Hence, the synchronous generator real power output can be represented as

$$P_e = P_{\max} \sin\delta \tag{2.3}$$

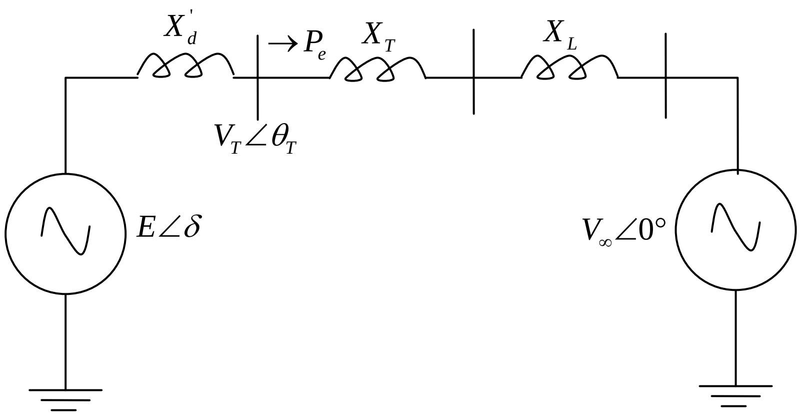


Fig. 2.1: Single line diagram of the SMIB system

Till now the equivalent electrical representation of the synchronous machine is discussed. The synchronous machine also has a mechanical system which has to be modeled. The prime mover gives mechanical energy to the generator rotor and in turn the generator converts the mechanical energy into electrical energy through magnetic coupling. The dynamics of a rotational mechanical system can be represented as

$$J \frac{d^2\theta_m}{dt^2} = T_m - T_e \tag{2.4}$$

where, J kg.m^2 is the inertia constant of the rotating machine. The mechanical input torque due to the prime mover is represented as T_m N.m and the electrical torque, acting against the mechanical input torque, is represented by T_e N.m. The angle θ_m is the mechanical angle of the rotor field axis with respect to the stator reference or fixed reference frame. As the rotor is continuously rotating at synchronous speed in steady state θ_m will also be continuously varying with respect to time. To make the angle θ_m constant in steady state we can measure this angle with respect to a synchronously rotating reference instead of a stationary reference. Hence, we can write

$$\boxed{\theta_m = \delta_m + \omega_{ms} t} \tag{2.5}$$

Where, δ_m is the angle between the rotor field axis and the reference axis rotating synchronously at ω_{ms} rps. If we differentiate (2.5) with respect to time we get

$$\frac{d\theta_m}{dt} = \frac{d\delta_m}{dt} + \omega_{ms} \tag{2.6}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \tag{2.7}$$

But, the rate of change of the rotor mechanical angle θ_m with respect to time is nothing but the speed of the rotor. Hence,

$$\omega_m = \frac{d\theta_m}{dt} \text{ rps} \tag{2.8}$$

Substituting, (2.8) in (2.6) we get

$$\frac{d\delta_m}{dt} = \omega_m - \omega_{ms} \tag{2.9}$$

Similarly, substituting (2.7) in (2.4) we get

$$\boxed{J\frac{d^2\delta_m}{dt^2} = T_m - T_e} \tag{2.10}$$

if we multiply with ω_m on both the side of (2.10) and noting that torque multiplied by speed gives power, we can write (2.10) as

$$\boxed{J\omega_m \frac{d^2\delta_m}{dt^2} = P_m - P_e} \tag{2.11}$$

Now multiply with the term $\frac{1}{2}\omega_{ms}$ on both the sides of (2.11) and divide the entire equation with the base MVA (S_B), in order to express the equation in per unit, lead to

$$\frac{\frac{1}{2}J\omega_m\omega_{ms}}{S_B}\frac{d^2\delta_m}{dt^2} = \frac{1}{2}\omega_{ms}\left(\frac{P_m}{S_B} - \frac{P_e}{S_B}\right) \tag{2.12}$$

Let us define a new parameter named as machine inertia constant

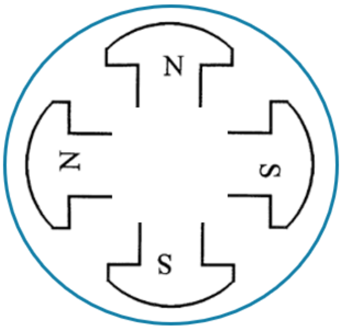
$$\boxed{H = \frac{\frac{1}{2}J\omega_{ms}{}^2}{\text{Base MVA}} = \frac{\frac{1}{2}J\omega_{ms}{}^2}{S_B}\ \frac{\text{MW.S}}{\text{MVA}}} \qquad \text{H = Kinetic Energy / MVA} \tag{2.13}$$

If we assume that on left hand side of (2.12) $\omega_m \cong \omega_{ms}$ as the variation of the speed, even during transients, from synchronous speed is quite less. This assumption does not mean that the speed of the rotor has reached the synchronous speed but instead $\frac{1}{2}J\omega_{ms}^2 \cong \frac{1}{2}J\omega_m\omega_{ms}$. With this assumption if we substitute (2.13) in (2.12) we get

$$H\frac{d^2\delta_m}{dt^2} = \frac{1}{2}\omega_{ms}\left(P_m - P_e\right) \quad \text{per unit} \tag{2.14}$$

δ_m and ω_{ms} are expressed in mechanical radians and mechanical radians per second, in order to convert them in to electrical radians and electrical radians per second respectively we have to take the number of poles (P) of the synchronous machine rotor into consideration. Hence, the electrical angle and electrical speed can be represented as

$$\delta = \frac{P}{2}\delta_m \ \text{elec.rad}$$
$$\omega_s = \frac{P}{2}\omega_{ms} \ \text{elec.rad/s} \tag{2.15}$$



1 pole (N-S) pair → 1 Mech Cycle = 1 Elec Cycle
2 poles (N-S) pair → 1 Mech Cycle = 2 Elec Cycle

Substituting (2.3) and (2.15) into (2.14) we get

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}\left(P_m - P_{\max}\sin\delta\right) \quad \text{per unit}$$
$$or \tag{2.16}$$
$$\frac{d^2\delta}{dt^2} = \frac{\pi f_s}{H}\left(P_m - P_{\max}\sin\delta\right) \quad \text{per unit } (f_s = 50 \text{ or } 60 \text{ Hz})$$

Equation (2.16) is called as swing equation. Equation (2.16), assuming all the parameters expressed in per units, can also be written as

$$\frac{d\delta}{dt} = (\omega - \omega_s)$$
$$\frac{d\omega}{dt} = \frac{\pi f_s}{H}\left(P_m - P_{\max}\sin\delta\right) \tag{2.17}$$

Where, $\omega = (P/2)\omega_m$. In can be observed from (2.17) that, if $P_m = P_{\max}\sin\delta$ then there will be no speed change and there will be no angle change. But, if $P_m \neq P_{\max}\sin\delta$ due to disturbance in the system then either the speed increase or decrease with respect to time. Let us take the case of $P_m > P_{\max}\sin\delta$, there is more input mechanical power than the electrical power output. In this case, as the energy has to be conserved difference between the input and output powers will lead to increase in the kinetic energy of the rotor and speed increases. Similarly, if $P_m < P_{\max}\sin\delta$ then, the input power is less than the required electrical power output. Again the balance power, to meet the load requirement, is drawn from the kinetic energy stored in the rotor due to which the rotor speed decreases.